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GEORGE C. MARSHALL SPACE FLIGHT CENTER

HUNTSVILLE, ALABAMA

# PARAMETRIC PERFORMANCE ANALYSIS OF LUNAR MISSIONS

## Part II: Lunar Descent

By Charles M. Akridge and Sam H. Harlin



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## ABSTRACT

The effect of thrust-to-weight ratios, and apocenter altitude on trajectory parameters has been investigated for descent to the lunar surface from parking orbit. A single stage with constant thrust directed against the velocity vector was used in all instances. Specific impulses of 300 and 420 sec were used for comparison.

The purpose of this report is to present trajectory parameters for the powered lunar landing maneuver from various lunar orbit altitudes. The results of the study show the variation of trajectory parameters for earth thrust-to-weight ratios from 0.2 to 0.5. These parameters include: true anomaly at initiation of powered descent; change in altitude, flight path angle and range angle during thrusting; burning time, characteristic velocity and velocity losses.

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FLIGHT OPERATIONS SECTION
ADVANCED FLIGHT SYSTEMS BRANCH
PROPULSION AND VEHICLE ENGINEERING DIVISION

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LIST OF ILLUSTRATIONS (Concluded)

## DEFINITION OF SYMBOLS

| Symbol | Definition |
|---|---|
| $F$ | Thrust, kp |
| $g$ | Gravitational acceleration, $m/sec^2$ |
| $g_e$ | Earth gravitational acceleration, 9.80665 $m/sec^2$ |
| $h$ | Altitude, km |
| $\Delta h$ | Altitude change, $h - h_o$, km |
| $I_{sp}$ | Specific impulse, sec |
| $m$ | Mass, $\frac{kp - sec^2}{m}$ |
| $r$ | Radius, km |
| $t$ | Time, sec |
| $\Delta t$ | Incremental time, sec |
| $S$ | Surface range, km |
| $V$ | Velocity, m/sec |
| $V^*$ | Comparative velocity, m/sec |
| $\Delta V$ | Stage characteristic velocity increment, m/sec |
| $W_o$ | Gross weight, kp |
| $F/W_o$ | Initial thrust-to-weight ratio (based on weight at sea level) |
| $W_s$ | Propellant weight, kp |
| $x$ | Range, km |
| $\alpha$ | Thrust vector orientation angle measured from the velocity to the thrust vector (positive down), deg |
| $\zeta$ | Propellant mass fraction, $W_s / W_o$ |
| $\gamma$ | Flight path angle from vertical, deg |
| $\Delta\gamma$ | Change in flight path angle, deg |
| $\mu_c$ | Gravitational constant for the moon, 4906 $km^3/sec^2$ |

## DEFINITION OF SYMBOLS (Concluded)

| Symbol | Definition |
|---|---|
| $\nu$ | True anomaly, deg |
| $\psi$ | Central angle from arbitrary reference, deg |
| $\Delta\psi$ | Central angle change, $\psi - \psi_o$, deg |
| Subscripts | |
| a | Apocenter |
| B | Burnout |
| e | Earth |
| ex | Exhaust |
| f | Final |
| id | Ideal |
| o | Initial |
| p | Pericenter |
| ☾ | Moon |

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## SUMMARY

The effect of thrust-to-weight ratios and apocenter altitude on trajectory parameters has been investigated for descent to the lunar surface from parking orbit. A single stage with constant thrust directed against the velocity vector was used in all instances. Specific impulses of 300 and 420 sec were used for comparison.

The results of the study show the variation of trajectory parameters for earth thrust-to-weight ratios from 0.2 to 0.5. These parameters include: true anomaly at initiation of powered descent; change in altitude, flight path angle, and range angle during thrusting; burning time, characteristic velocity, and velocity losses.

## SECTION I. INTRODUCTION

Orbital operations are of great interest in lunar exploration because of requirements imposed by the landing site, energy and tracking. Preliminary analysis of lunar missions requires a rapid method of sufficient accuracy for determining trajectory parameters.

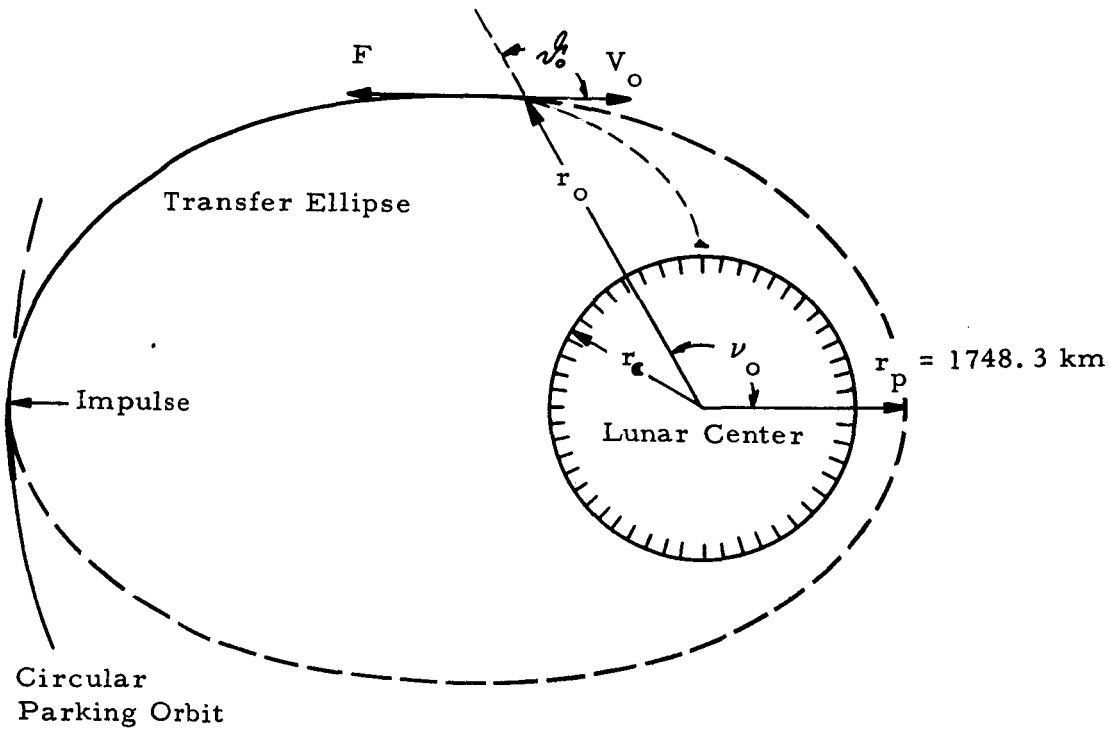
The purpose of this report is to present trajectory parameters for the powered lunar landing maneuver from various lunar orbit altitudes. The parameters presented are: velocity, altitude change,

surface range, change in flight path angle, change in central angle, true anomaly, burning time, propellant mass fraction and velocity losses.

The approach used was to apply an impulse in the parking orbit to attain a transfer ellipse with a 10-kilometer pericenter altitude. At the proper point on the transfer ellipse, powered braking was initiated to arrive at a 300-meter altitude with the zero velocity. The equations of motion were integrated on a RECOMP II computer, using Runge-Kutta numerical integration.

## SECTION II. ANALYSIS

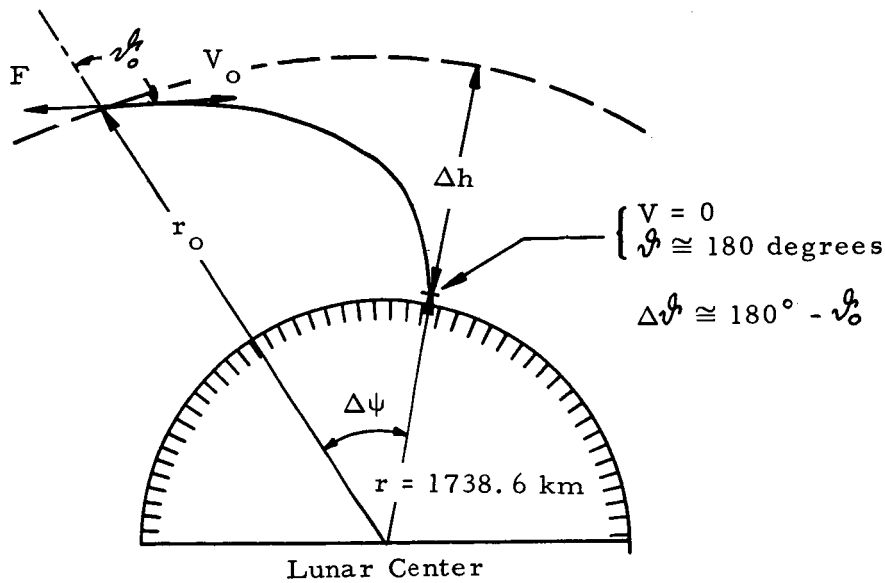
One method of landing on the lunar surface is to direct the thrust against the velocity vector ($\alpha = 180°$) and follow a gravity turn path. The sketch below illustrates the conditions at initiation of burning.




First,, an impulse (as calculated from FIG 1) was applied against the velocity vector to attain the transfer ellipse. The pericenter altitude of the transfer ellipse was taken as 10 km to insure against collision

with the surface if powered descent was not initiated. This pericenter altitude was selected because of surface irregularities, guidance errors, retro-thrust errors, etc. From a performance view, it is preferable to expend the energy at as low an altitude as possible.

The initial true anomaly (measured from the undisturbed Keplerian pericenter radius) varies with thrust-to-weight ratio and parking orbit altitude. It is seen that as parking orbit altitude decreases the initial true anomaly will increase (to a maximum of 180°).




The sketch above shows the initial and final trajectory parameters, as well as change in flight path angle, altitude, and central range angle. The line of sight and angular range can be determined from FIG 2, using initial conditions. The equations of motion were numerically integrated from initial conditions to 300-meters above the surface. The initial conditions were found by iteration to give the desired end conditions.

The following point mass, planar equations of motion were used with $\alpha = 180°$ in all instances.

$$\dot{V} = \frac{F \cos \alpha}{m} - \frac{g_c r_c^2}{r^2} \cos \gamma \tag{1}$$

$$V\dot{\gamma} = \frac{F \sin \alpha}{m} + \left[ \frac{g_c r_c^2}{r^2} - \frac{V^2}{r} \right] \sin \gamma \tag{2}$$

$$\dot{r} = V \cos \gamma \tag{3}$$

$$\dot{\psi} = \frac{V \sin \gamma}{r} \tag{4}$$

where

$$m = m_o + \int \dot{m}\, dt \tag{5}$$

and

$$\dot{m} = - \frac{F}{V_{ex}} \tag{6}$$

The velocity and flight path angle may be obtained by integrating the equation of motion

$$V = \int \dot{V}\, dt \tag{7}$$

$$\gamma = \int \dot{\gamma}\, dt \tag{8}$$

The range and pericenter altitude can then be calculated by the relations

$$x = \int \frac{r_c}{r} v \sin \gamma\, dt \tag{9}$$

$$h = h_o + \int \dot{r}\, dt \tag{10}$$

and the central angle

$$\psi = \int \frac{\dot{x}}{r_c} dt \tag{11}$$

and surface range

$$S = r_c \Delta\psi \tag{12}$$

The initial weight of the vehicle is

$$W_o = W_c + W_s \tag{13}$$

The undisturbed Keplerian true anomaly at initiation of powered descent is

$$\nu_o = \cos^{-1}\left[\frac{(V_a r_a)^2/\mu_c - r_o}{r_o\sqrt{1 - \frac{V_o^2 r_o}{\mu_c}\left(2 - \frac{V_o^2 r_o}{\mu_c}\right)\sin^2\gamma_o}}\right] \tag{14}$$

The velocity expended by the vehicle is the characteristic velocity, or

$$\Delta V = V_{ex} \ln\left(\frac{1}{1-\zeta}\right) \tag{15}$$

and may be determined from FIG 3. Consequently, the velocity losses are the difference between the characteristic velocity and the change in comparative velocity, or

$$\Delta V_{loss} = \Delta V - \Delta V^* \tag{16}$$

where the comparative velocity is

$$V^* = \sqrt{V_o^2 + 2\mu_c\left(\frac{1}{r} - \frac{1}{r_o}\right)} \tag{17}$$

and the change in comparative velocity during descent from $r = r_o$ to $r = r_f$ is

$$\Delta V^* = \sqrt{V_o^2 + 2\mu_c\left(\frac{1}{r_f} - \frac{1}{r_o}\right)} - V_f \tag{18}$$

By substituting in original equation for velocity losses

$$\Delta V_{loss} = V_{ex} \ln\left(\frac{1}{1-\zeta}\right) - \left[\sqrt{V_o^2 + 2\mu_c\left(\frac{1}{r_f} - \frac{1}{r_o}\right)} - V_f\right] \tag{19}$$

## SECTION III. ASSUMPTIONS

A summary of the basic assumptions used in this analysis follows:

1. Deceleration of a single stage from a transfer ellipse to a 300 meter (hover) altitude and zero velocity using constant thrust directed against the velocity vector.

2. Initial parking orbit altitudes

   h = 50 km

   h = 100 km

   h = 200 km

   h = 300 km

3. For comparison, specific impulses of 300 sec and 420 sec were used.

4. The initial earth thrust-to-weight ratio for a chemical stage was varied parametrically from 0.2 to 0.5.

5. Mean spherical moon

   $\mu_{\mathbb{C}} = 4906 \text{ km}^3/\text{sec}^2$.

   $r_{\mathbb{C}} = 1738.3$ km

## SECTION IV. RESULTS AND CONCLUSIONS

The results are shown in FIG 4 through 13. For convenience, most variables are shown as a function of initial vehicle thrust-to-weight ratio.

The characteristic impulse velocity, shown in FIG 4, is the velocity necessary to attain a transfer ellipse from a circular parking orbit altitude. The apocenter altitude of the transfer ellipse is assumed to be circular parking orbit altitude and the pericenter altitude is 10 km in all instances. Since losses were negligible, an impulsive velocity was assumed.

The characteristic velocity for descent to 300-meters above the lunar surface is shown in FIG 5a and 5b for specific impulses of 300 sec and 420 sec, respectively. The corresponding propellant mass fraction is shown in FIG 6a and 6b.

FIG 7a and 7b show the altitude change, $\Delta h$, during powered descent for specific impulses of 300 sec and 420 sec, respectively. A limit is reached, for a gravity turn ($\alpha = 180°$), when the altitude change becomes equal to the apocenter altitude. This limit determines the minimum initial thrust-to-weight ratio for continuous burn from parking orbit to 300 m above the surface. This limit is reflected in the change in flight path angle, $\Delta\gamma$, FIG 8a and 8b when $\Delta\gamma = 90°$.

The change in central angle, $\Delta\psi$ (angular range), during powered descent is given in FIG 9a and 9b. The product of $\Delta\psi$ (in radians) and the mean lunar radius is the surface range and is shown in FIG 10a and 10b. The limit shown for $\Delta\psi$ and S is reached when powered descent is initiated at apocenter.

The true anomaly, $\nu_o$, shown in FIG 11a and 11b, is measured from the undisturbed Keplerian pericenter radius to the radius at initiation of powered descent. True anomaly equals 180° for the limit of minimum thrust-to-weight ratio. Burning times for the powered descent are given in FIG 12a, 12b, 12c, and 12d. The true gravity losses are shown in FIG 13a and 13b. Burning times and gravity losses are a maximum in the limiting case.

# SECTION V. GRAPHIC PRESENTATION

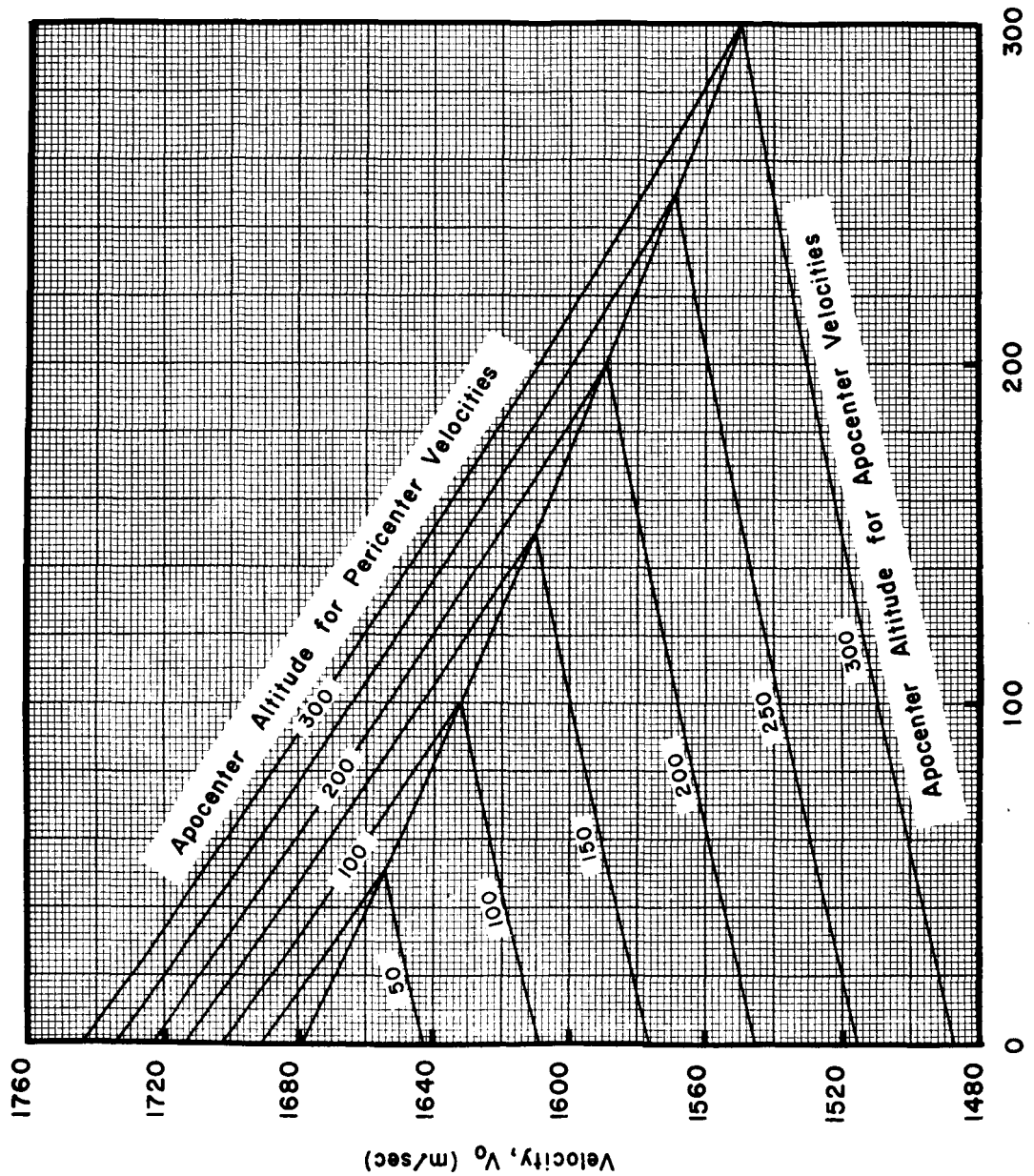



FIGURE 1. APOCENTER AND PERICENTER VELOCITIES VERSUS APOCENTER AND PERICENTER ALTITUDES

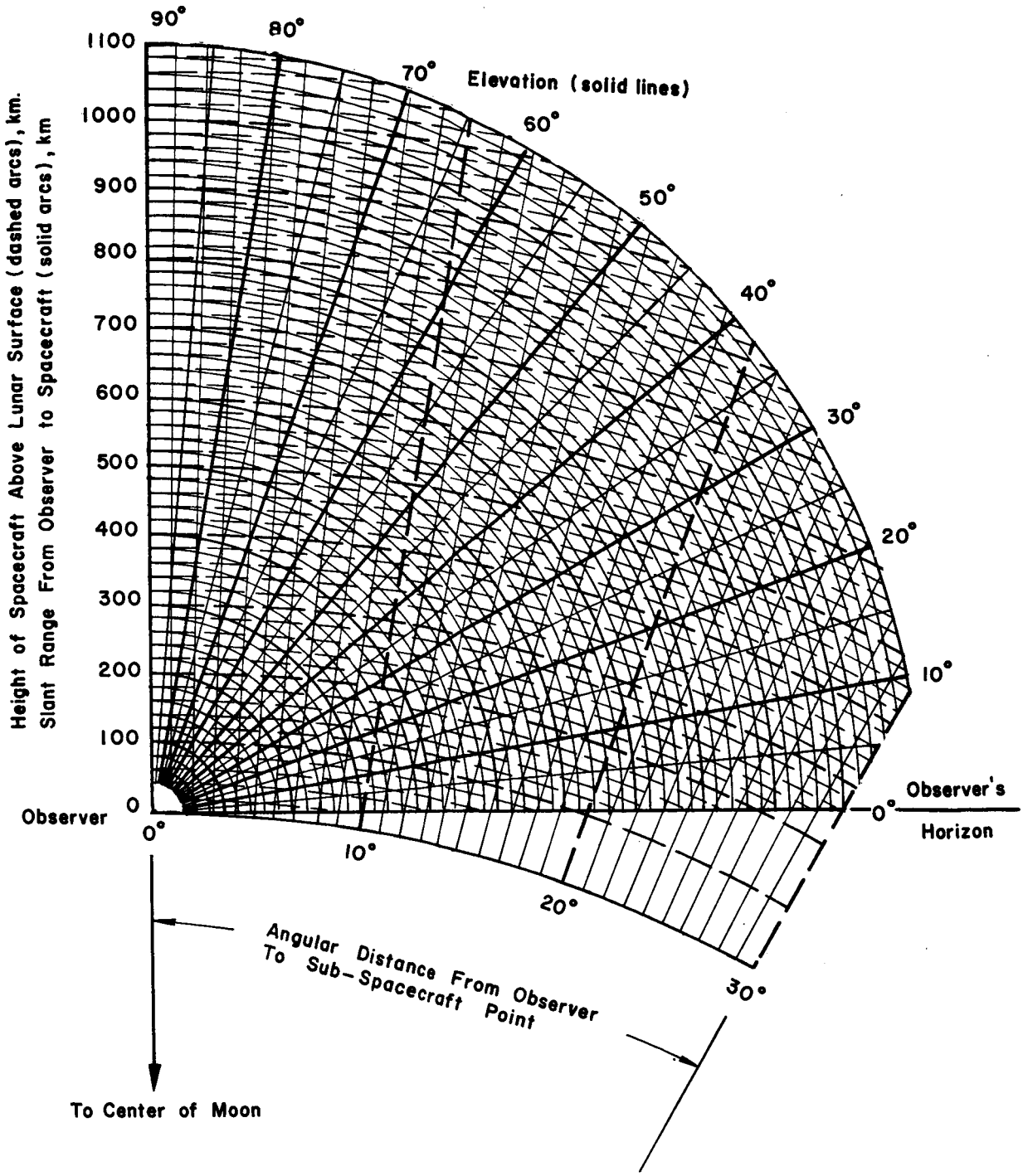



FIGURE 2. CHART FOR DETERMINING ELEVATION AND SLANT RANGE OF LUNAR SPACECRAFT (0 to 1100 km)

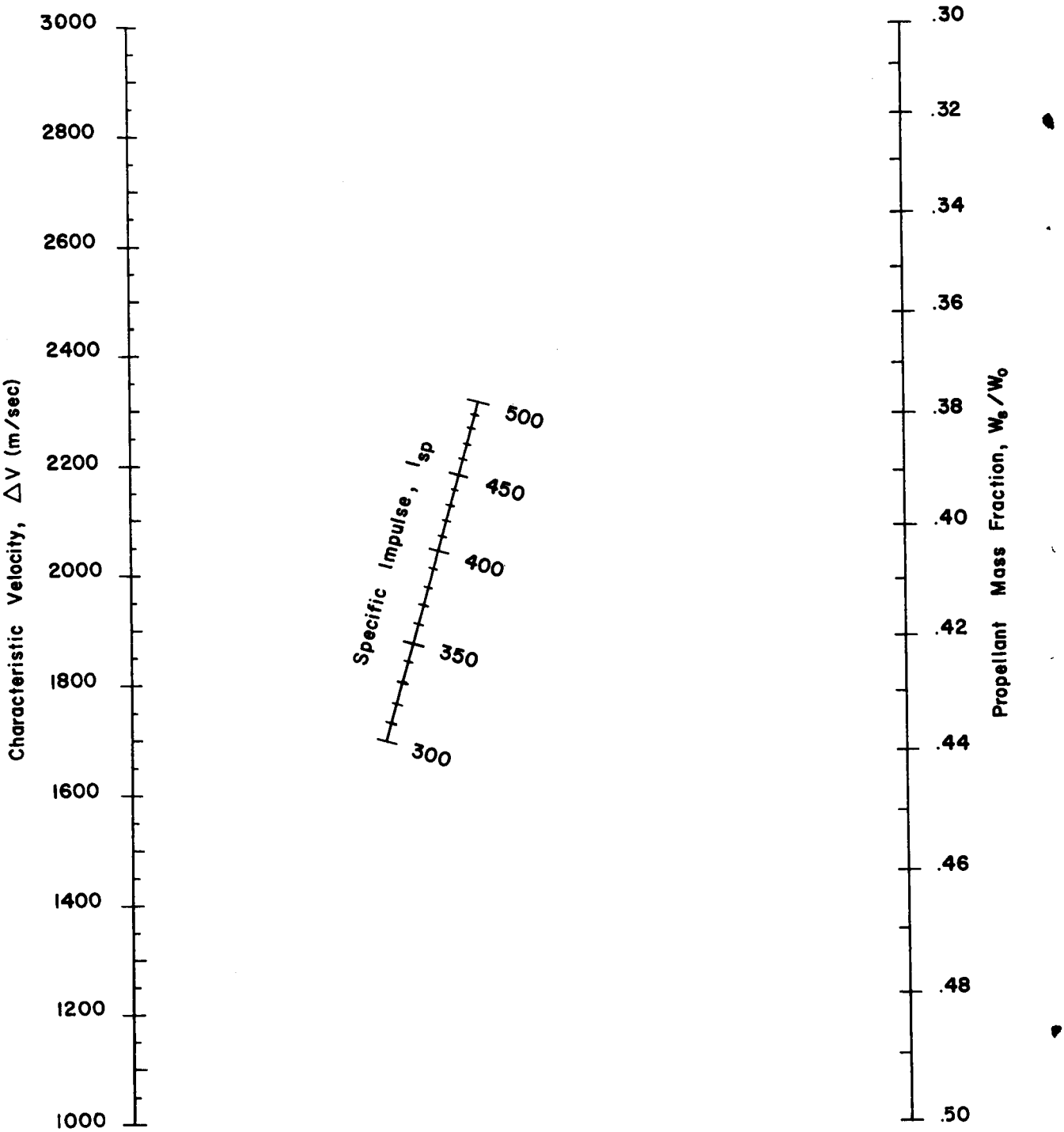



FIGURE 3. CHARACTERISTIC VELOCITY AS A FUNCTION OF PROPELLANT MASS FRACTION AND SPECIFIC IMPULSE

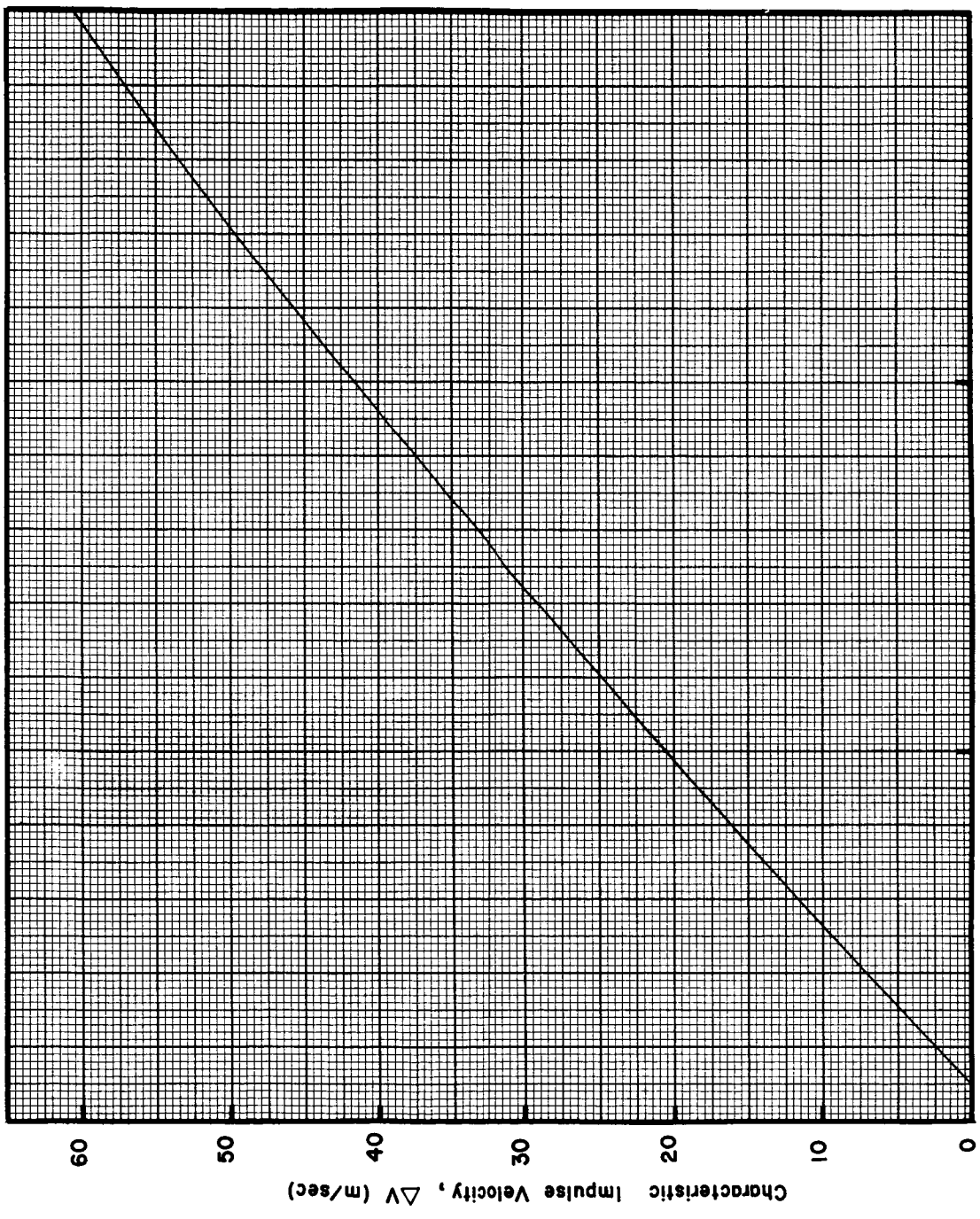



FIGURE 4. CHARACTERISTIC IMPULSE VELOCITY TO ATTAIN TRANSFER ELLIPSE WITH $r_p$ = 1748.3 km

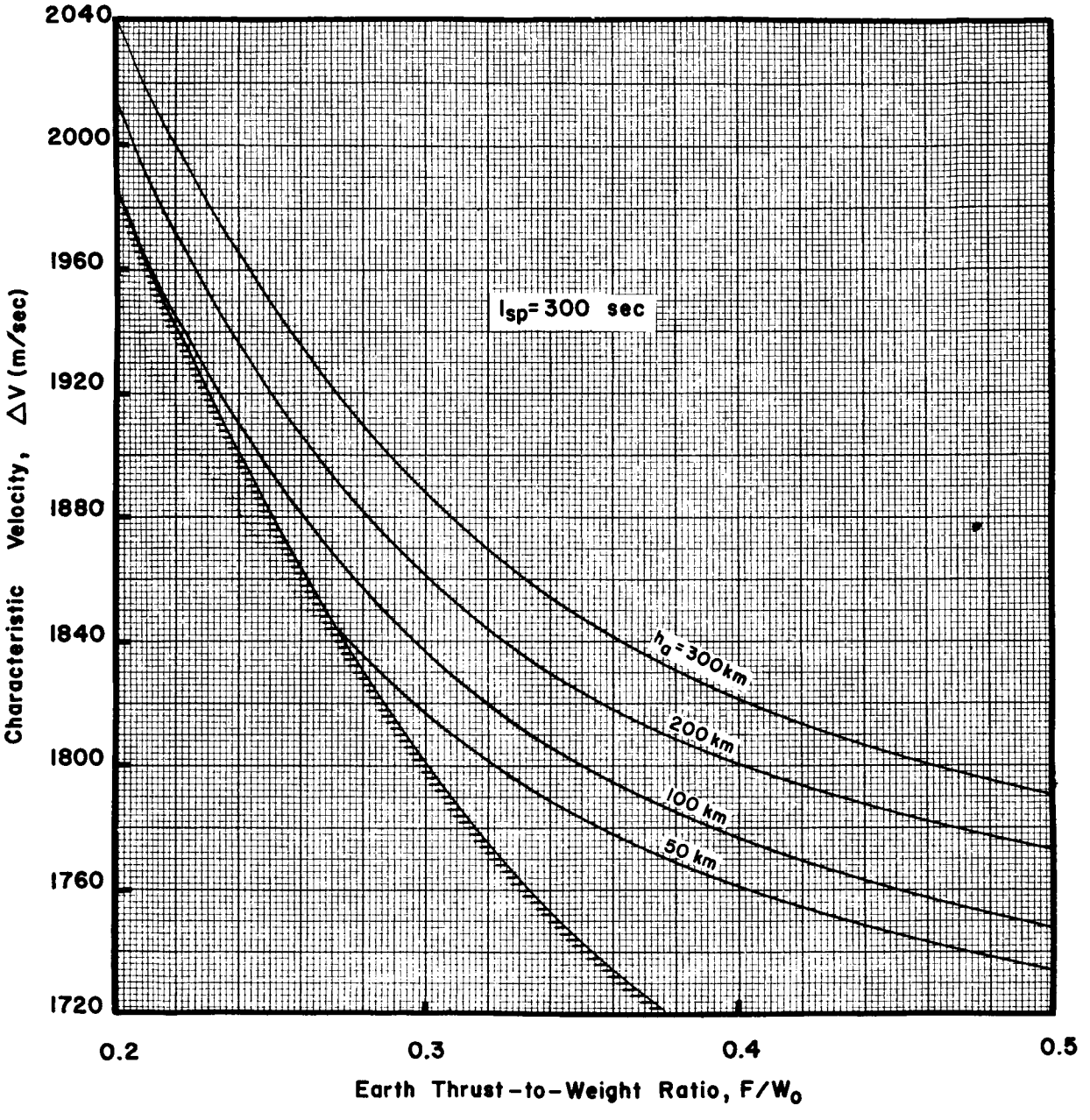



FIGURE 5a. CHARACTERISTIC VELOCITY FOR LUNAR DESCENT - $I_{sp}$ = 300 sec.

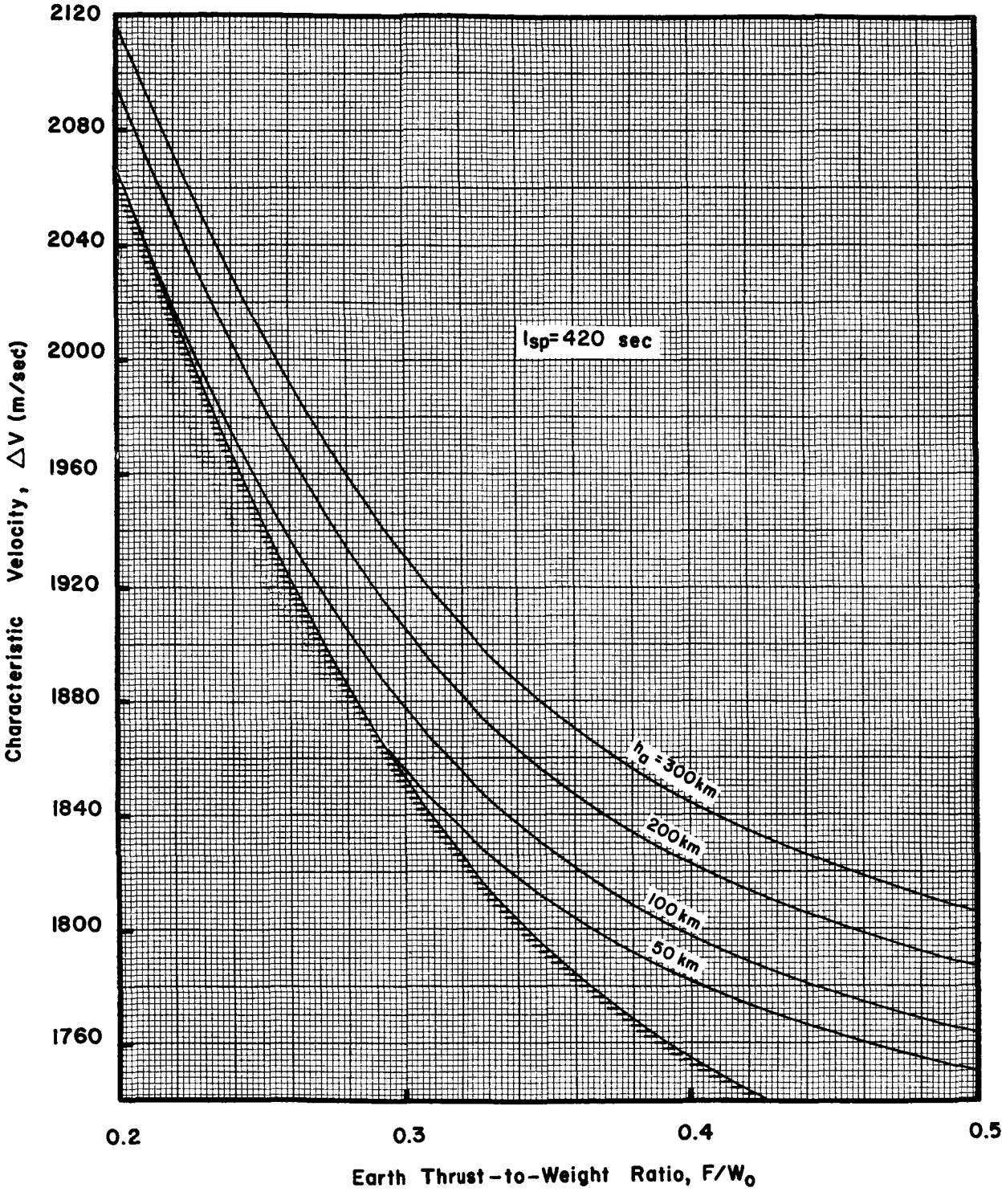



FIGURE 5b. CHARACTERISTIC VELOCITY FOR LUNAR DESCENT - $I_{sp}$ = 420 sec.

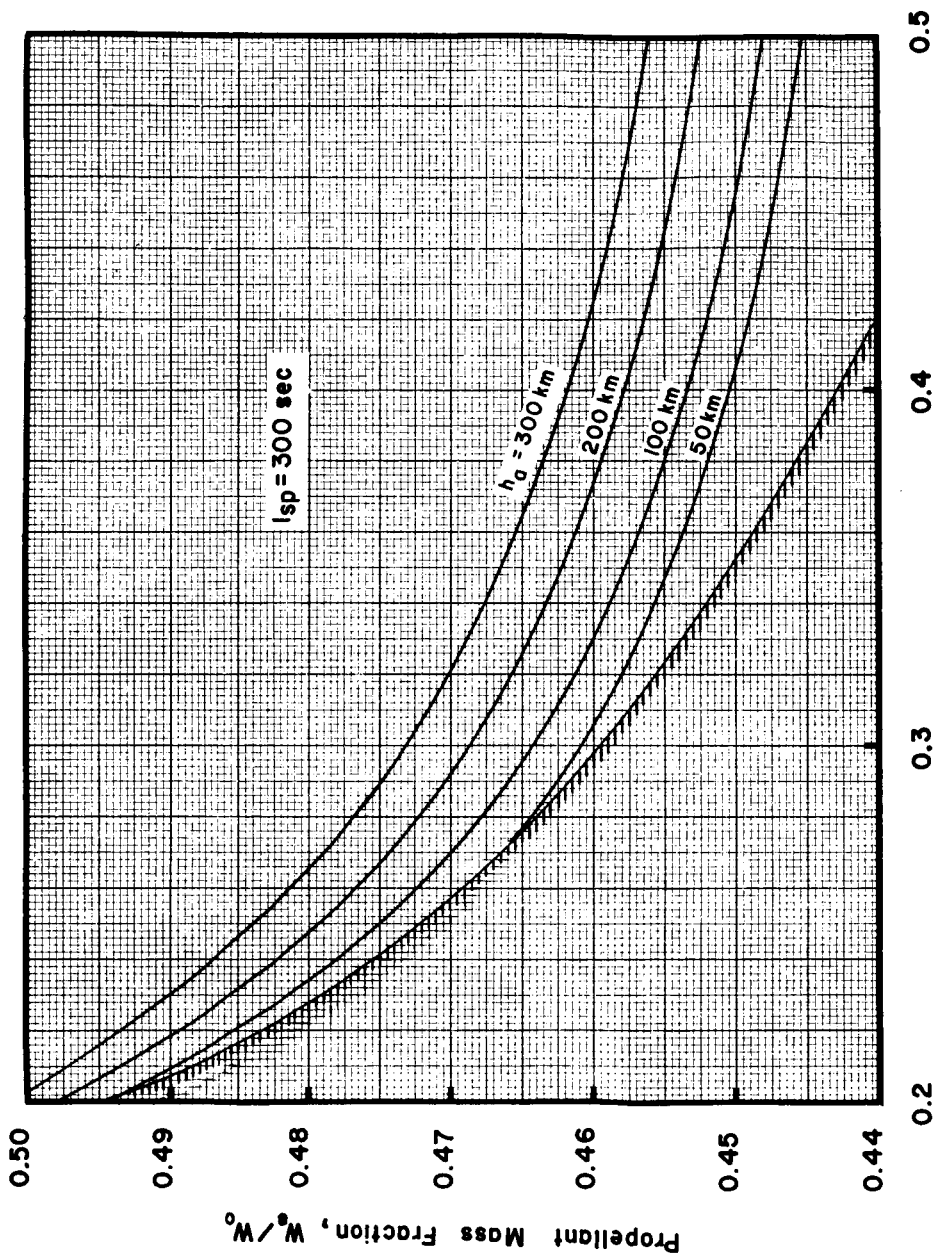



FIGURE 6a. PROPELLANT MASS FRACTION FOR LUNAR DESCENT - $I_{sp}$ = 300 sec.

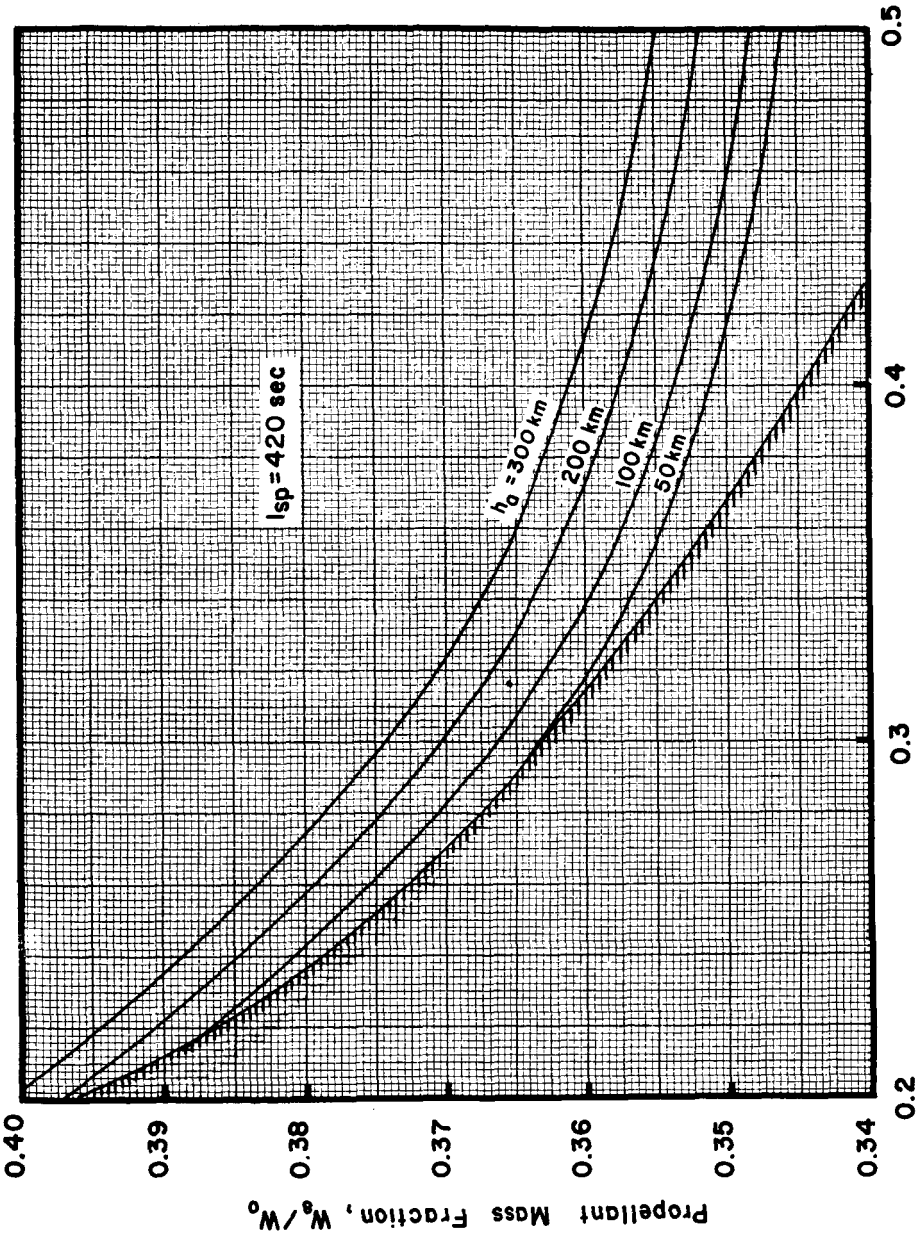



FIGURE 6b. PROPELLANT MASS FRACTION FOR LUNAR DESCENT - $I_{sp}$ = 420 sec.

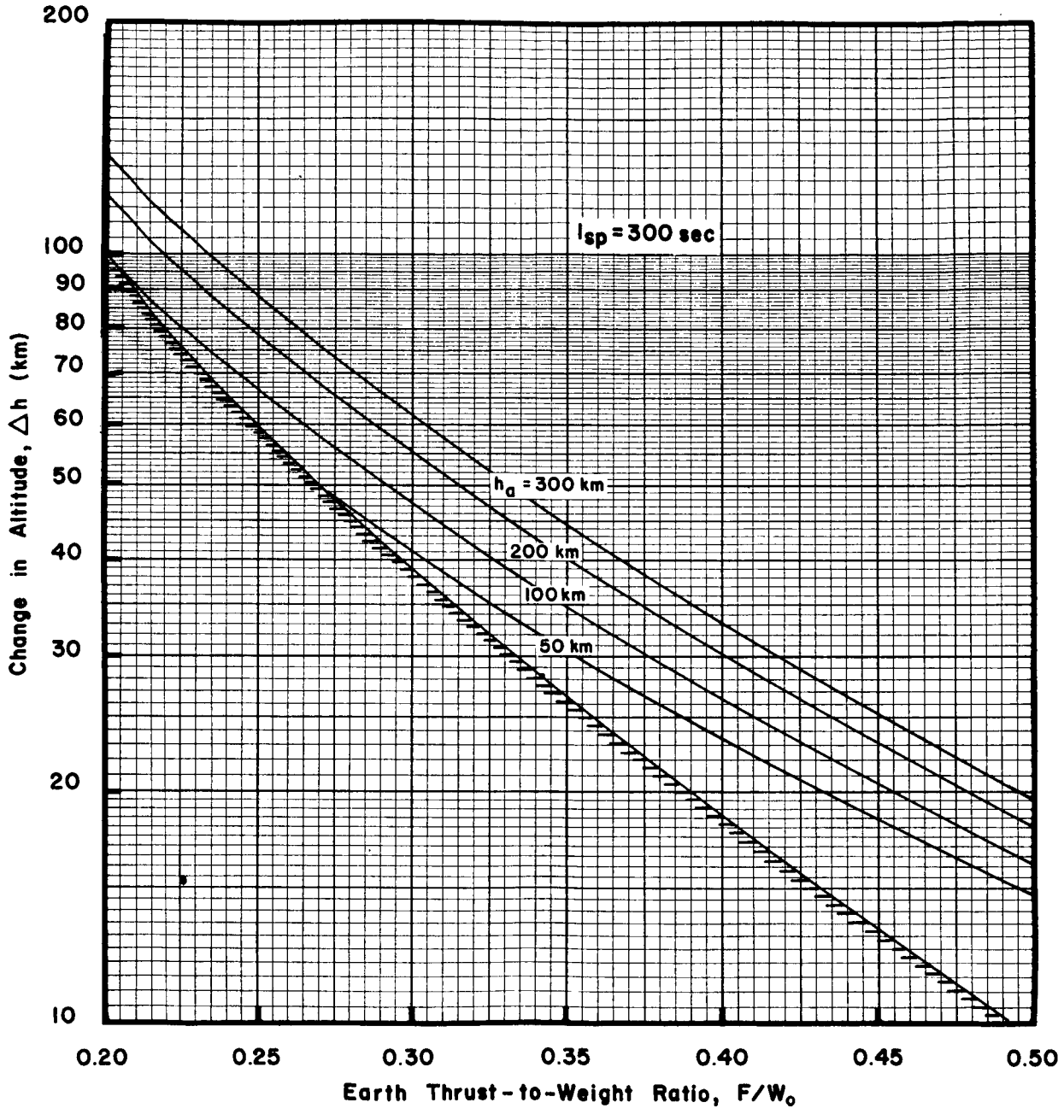



FIGURE 7a. CHANGE IN ALTITUDE FOR LUNAR DESCENT - $I_{sp}$ = 300 sec.

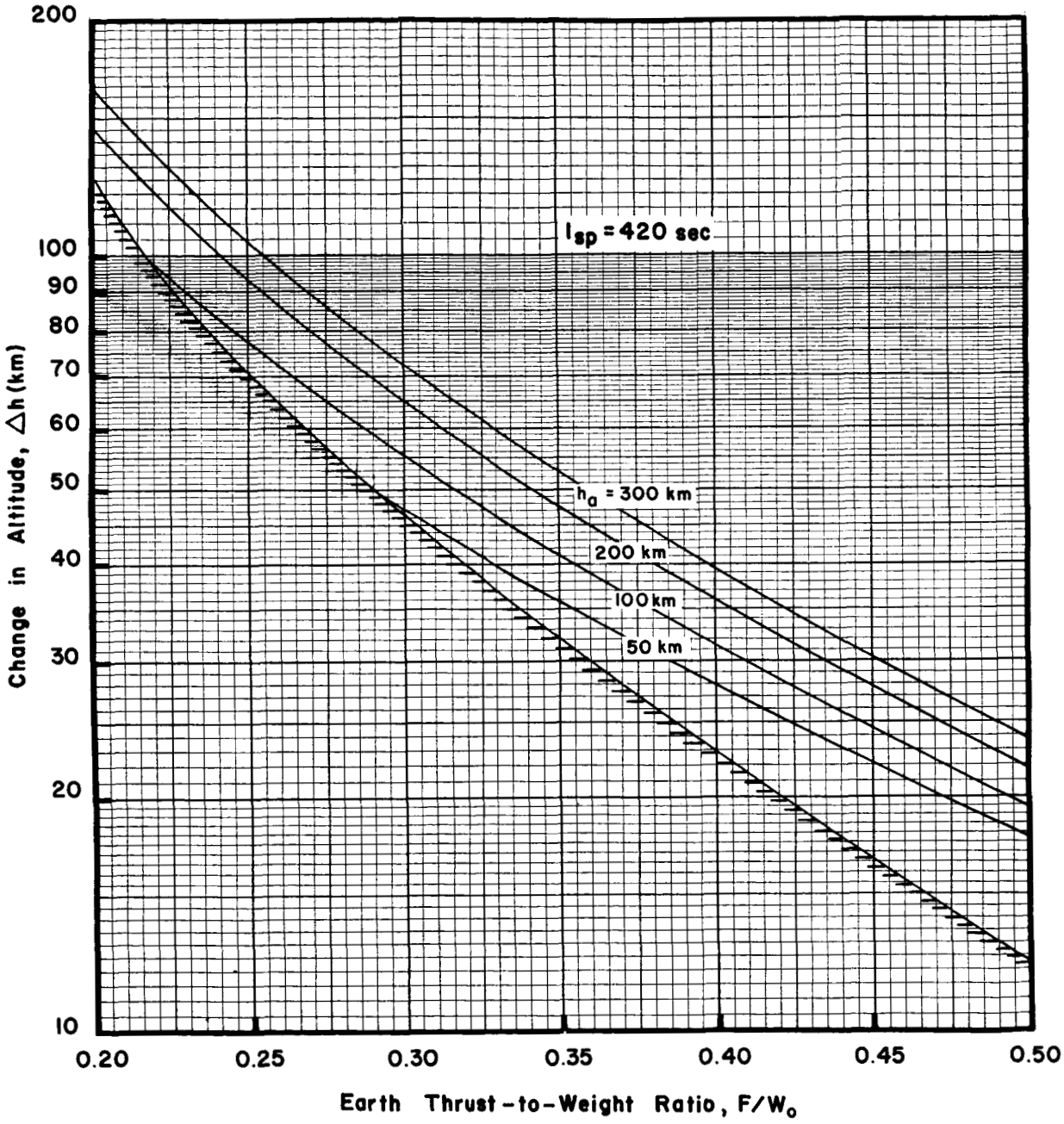



FIGURE 7b. CHANGE IN ALTITUDE FOR LUNAR DESCENT - $I_{sp}$ = 420 sec.

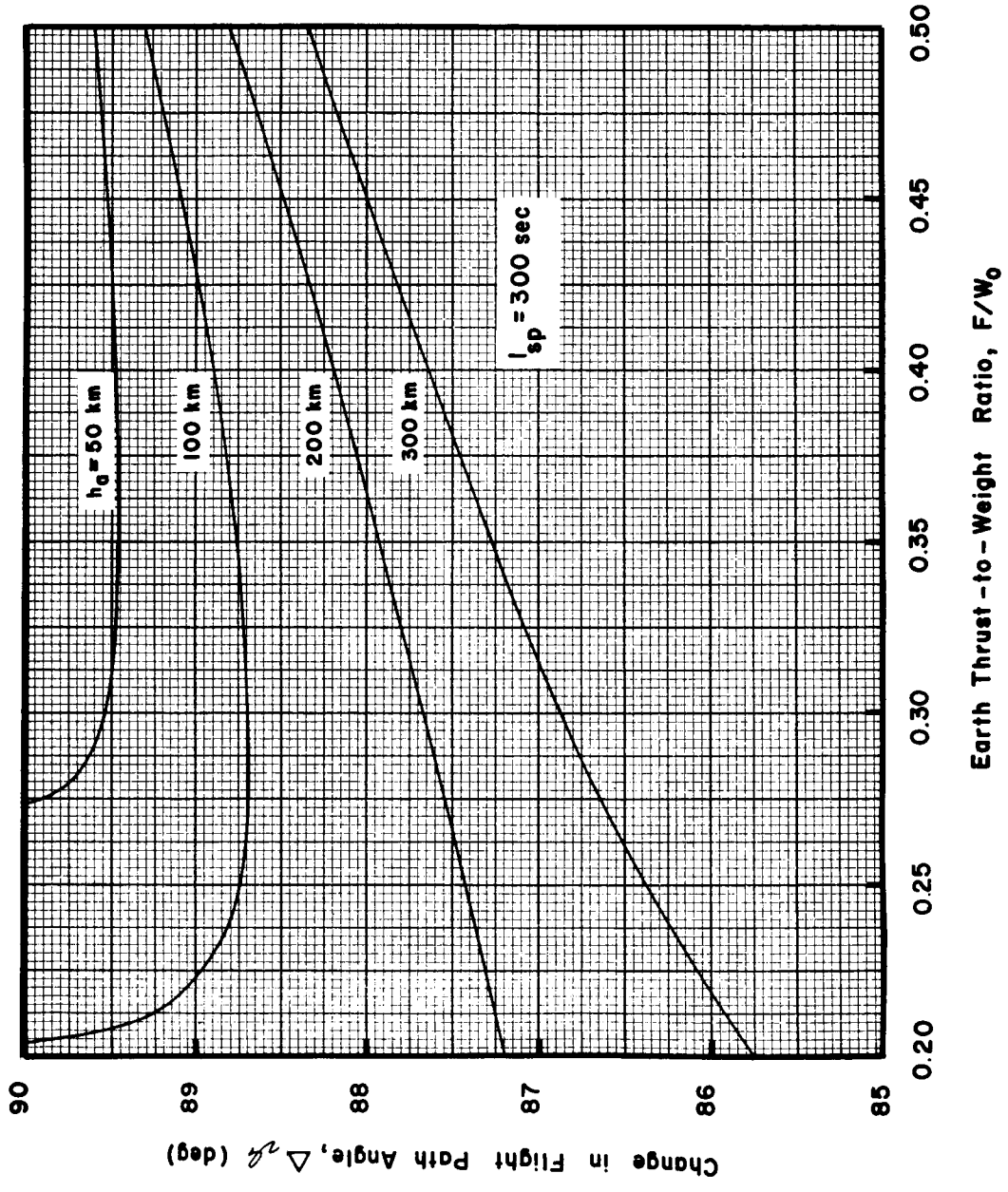



FIGURE 8a. CHANGE IN FLIGHT PATH ANGLE FOR LUNAR DESCENT - $I_{sp}$ = 300 sec.

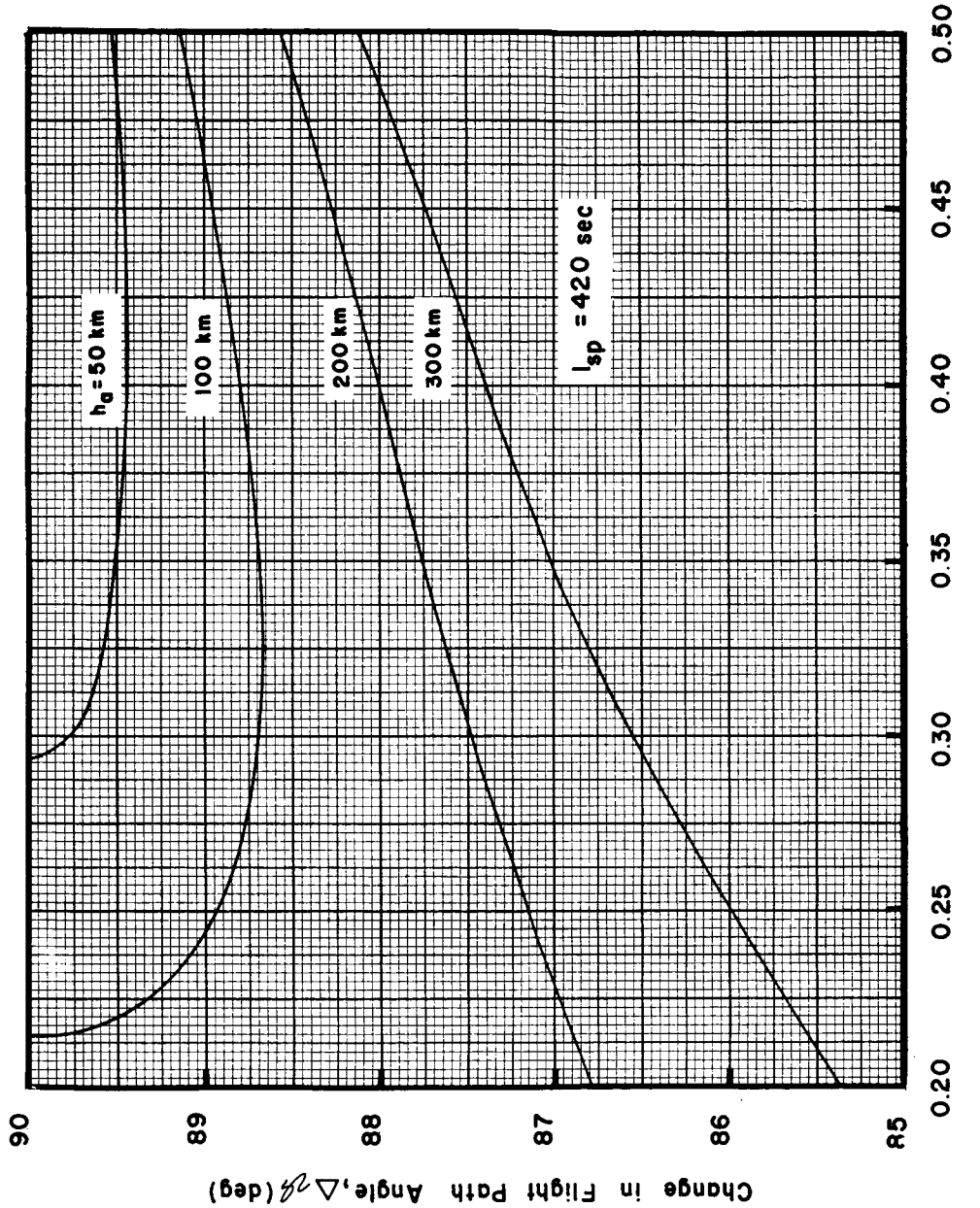



FIGURE 8b. CHANGE IN FLIGHT PATH ANGLE FOR LUNAR DESCENT - $I_{sp}$ = 420 sec.

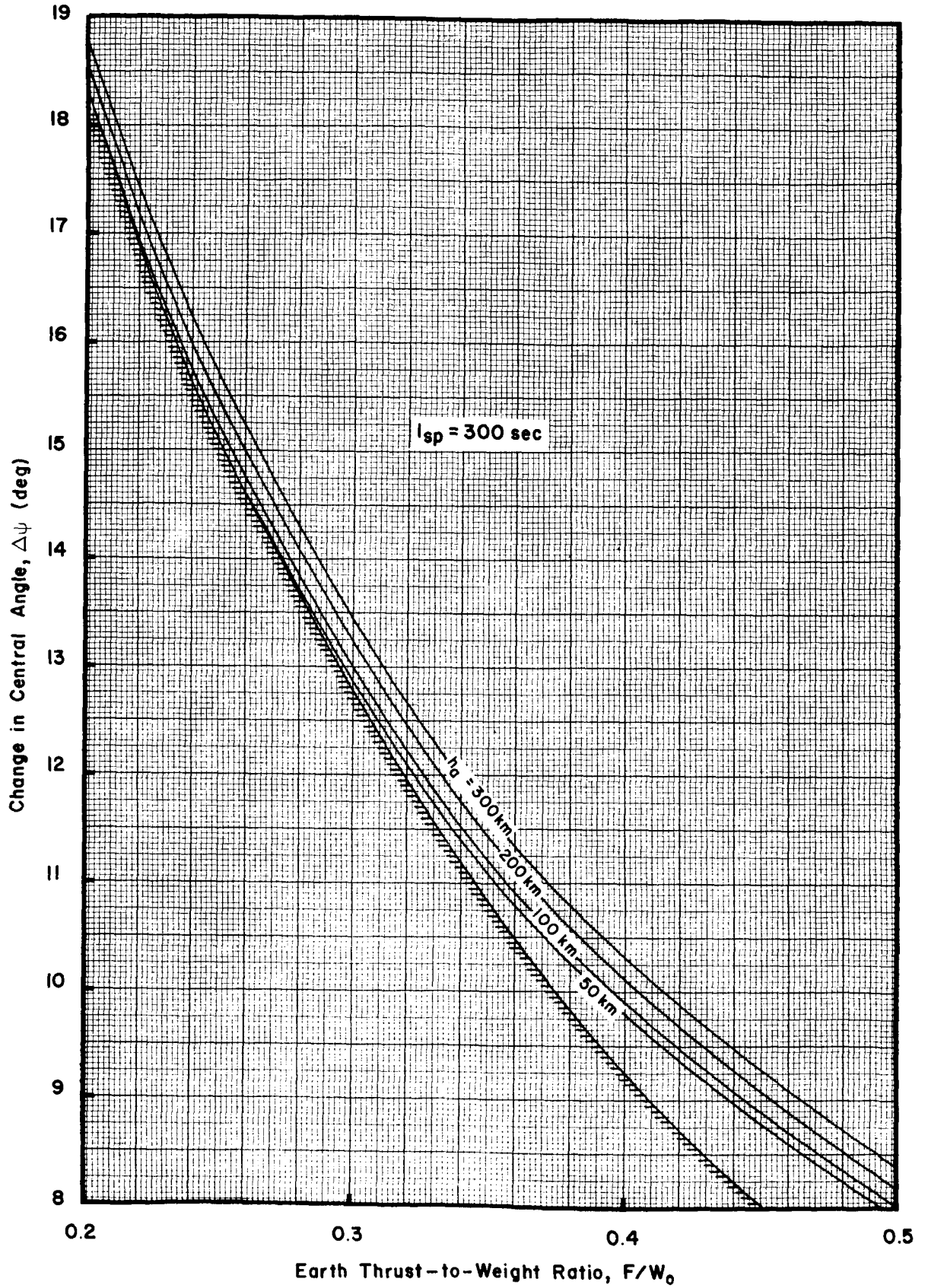



FIGURE 9a. CHANGE IN CENTRAL ANGLE FOR LUNAR DESCENT - $I_{sp}$ = 300 sec.

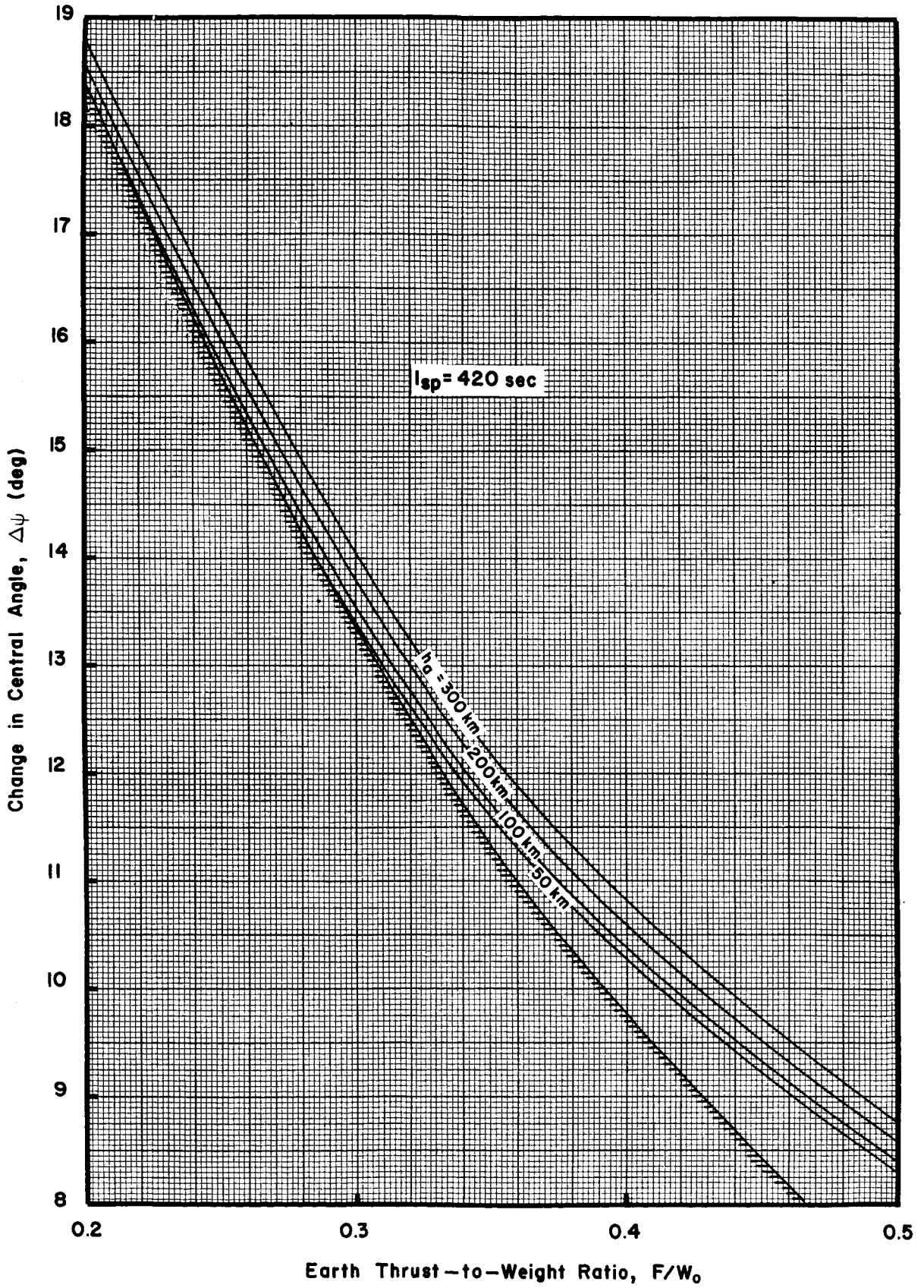



FIGURE 9b. CHANGE IN CENTRAL ANGLE FOR LUNAR DESCENT - $I_{sp}$ = 420 sec.

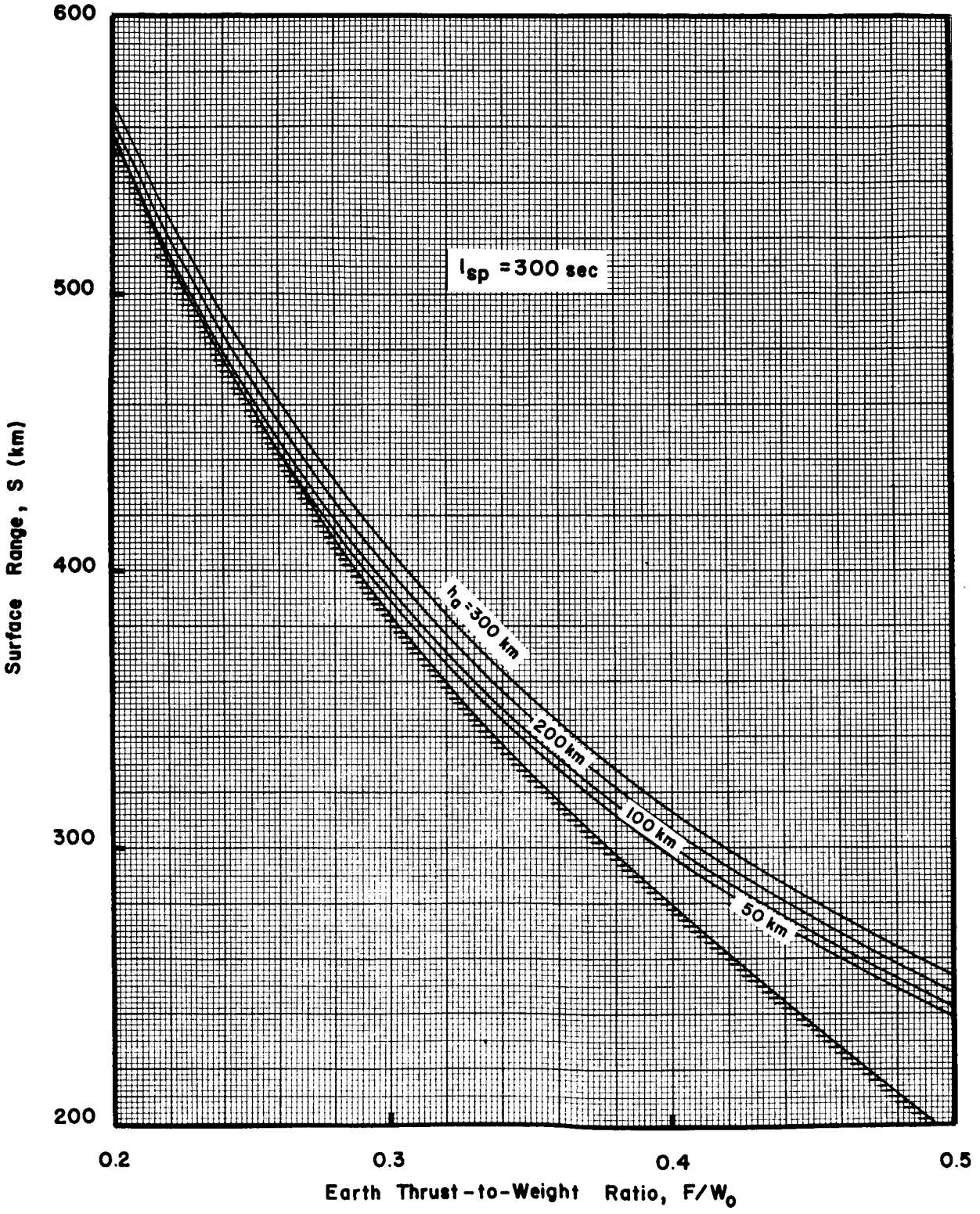



FIGURE 10a. SURFACE RANGE FOR LUNAR DESCENT - $I_{sp}$ = 300 sec.

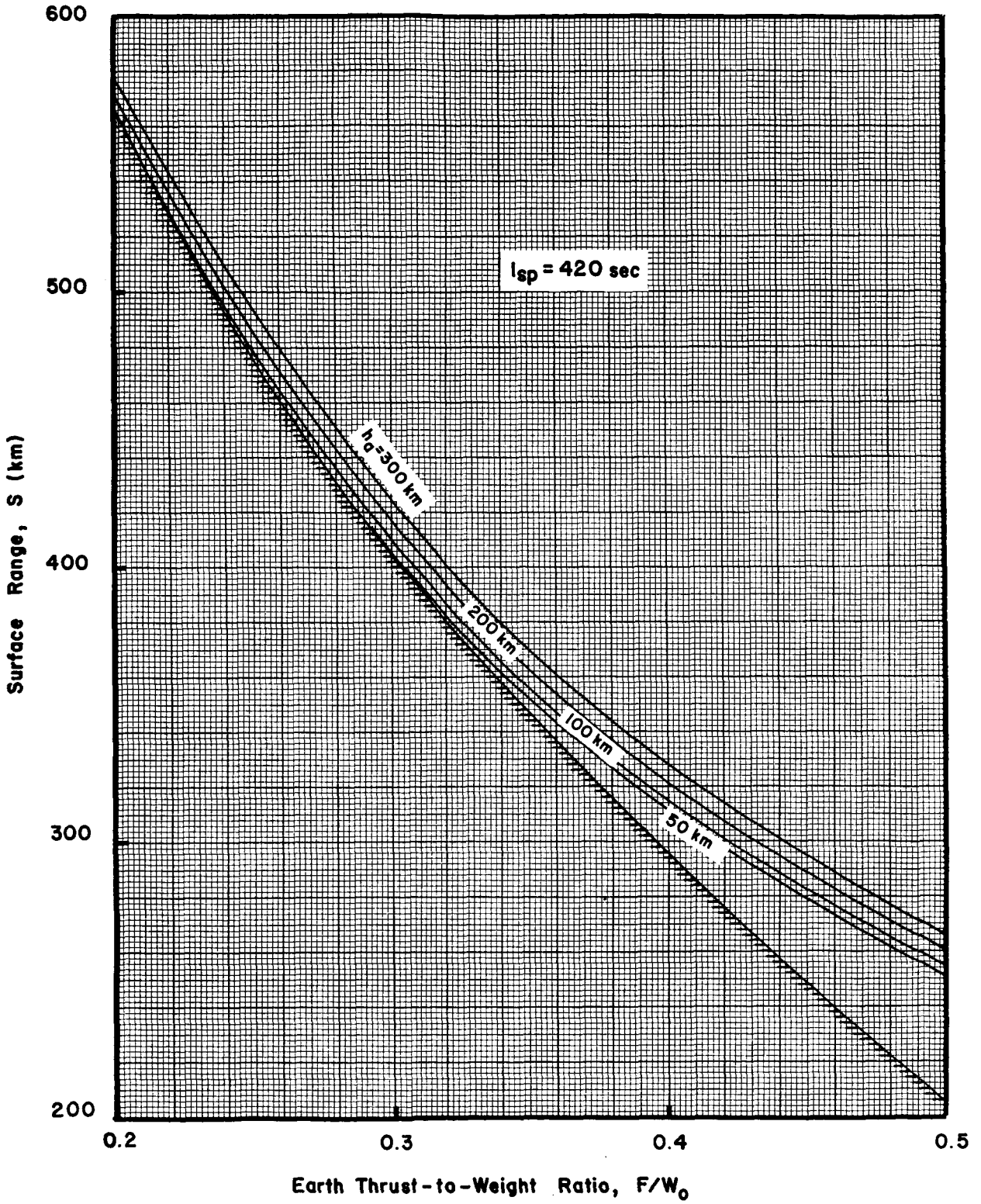



FIGURE 10b. SURFACE RANGE FOR LUNAR DESCENT - $I_{sp}$ = 420 sec.

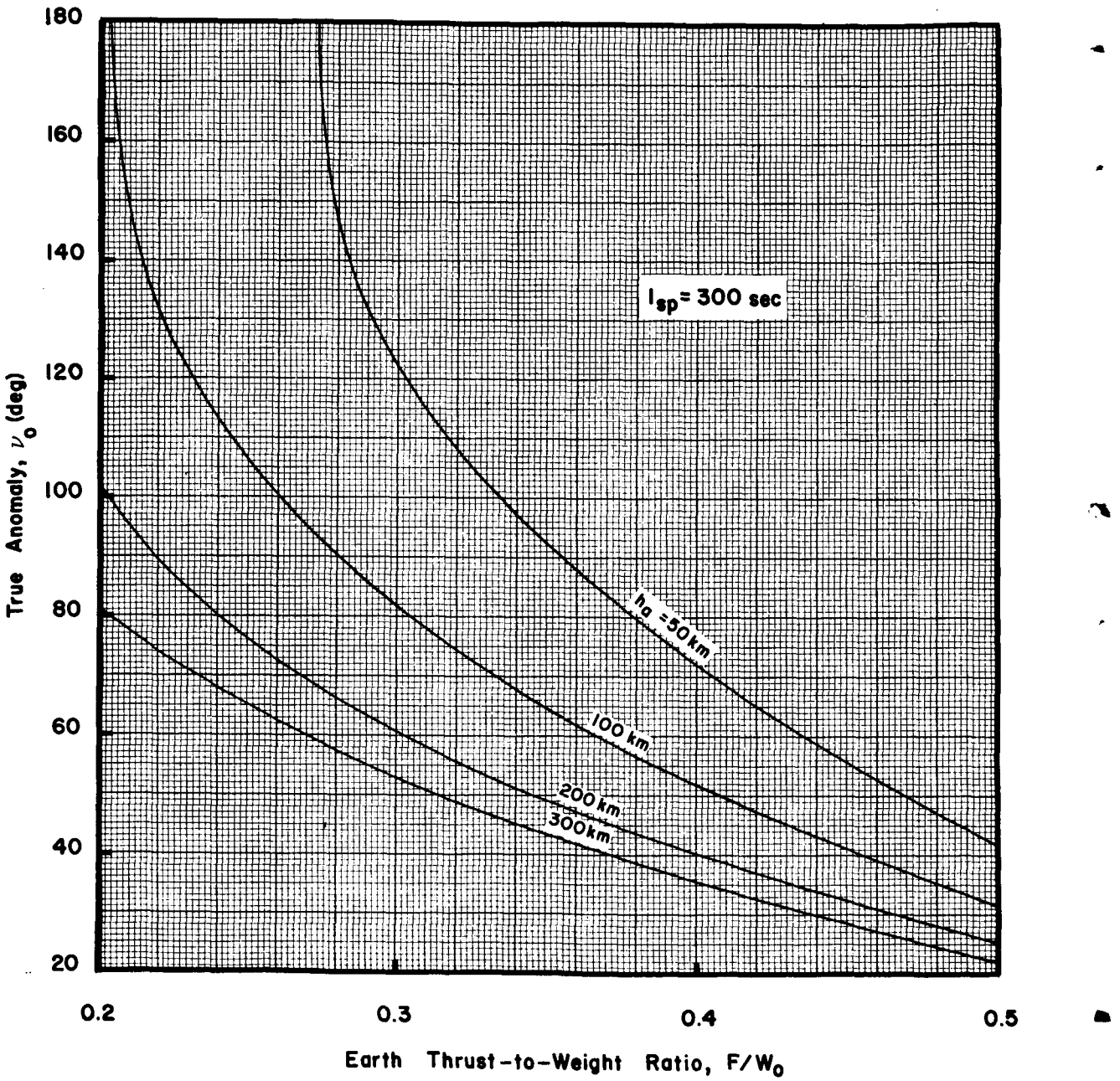



FIGURE 11a. TRUE ANOMALY AT INITIATION OF LUNAR DESCENT - $I_{sp}$ = 300 sec.

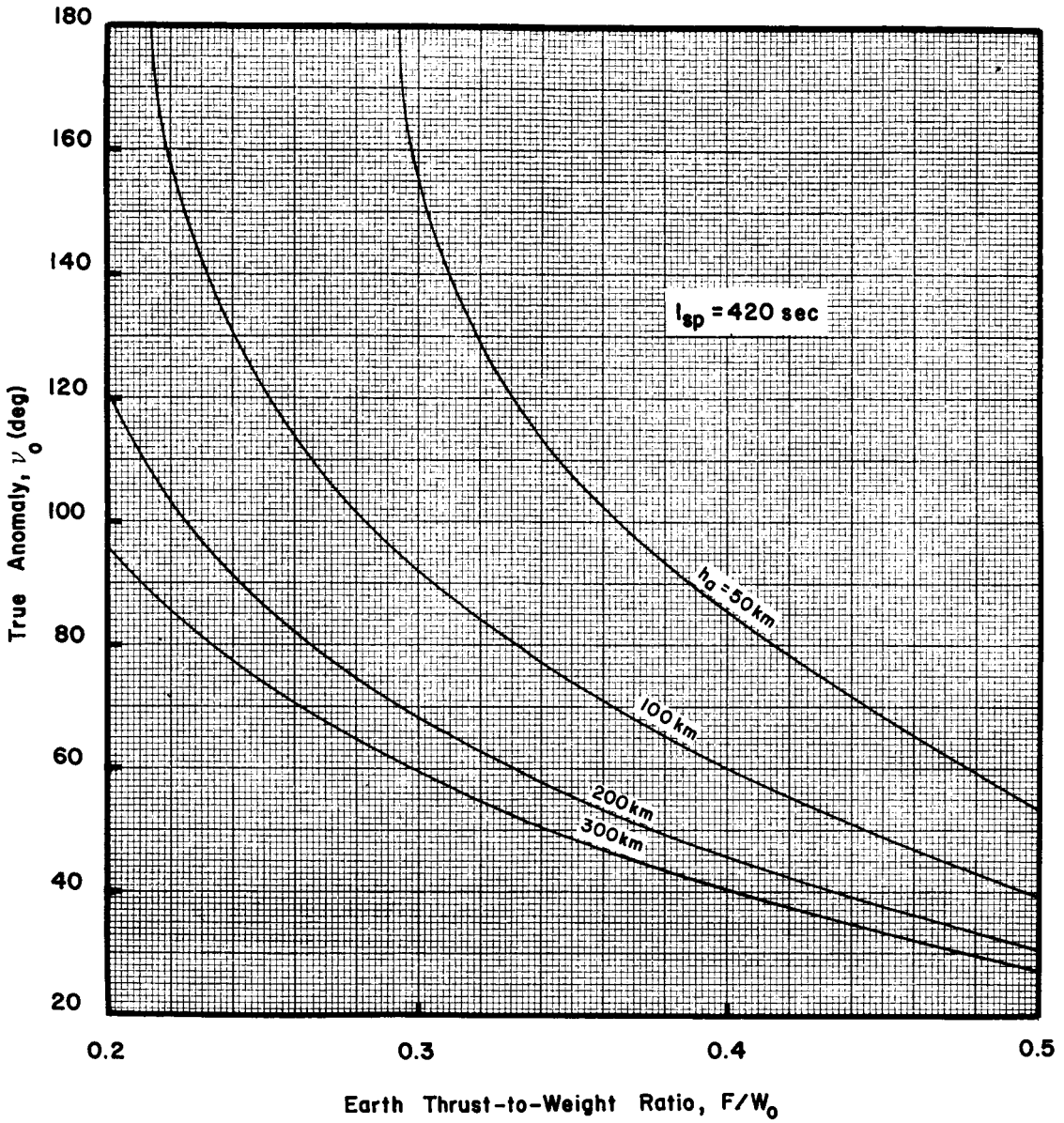



FIGURE 11b. TRUE ANOMALY AT INITIATION OF LUNAR DESCENT - $I_{sp}$ = 420 sec.

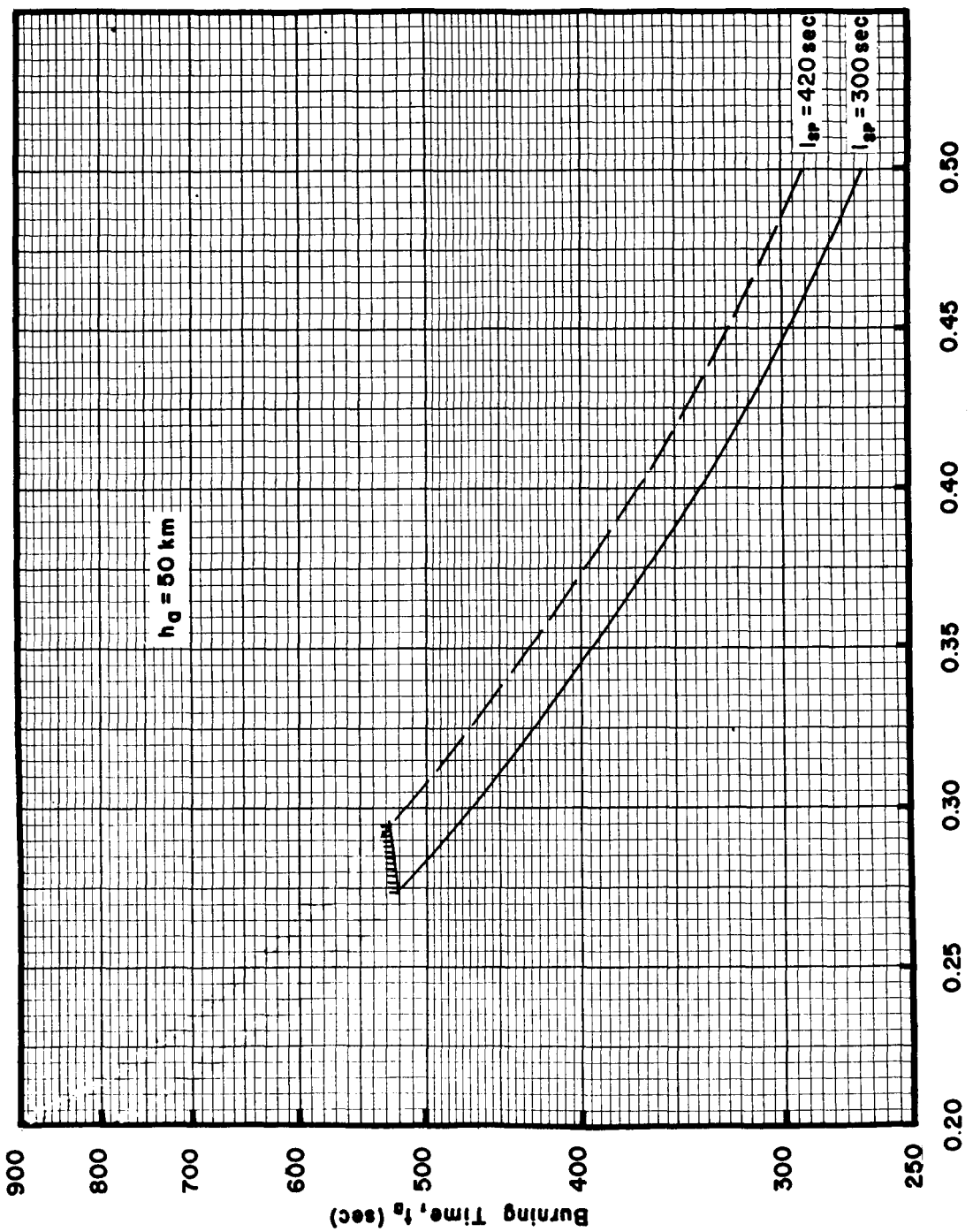



FIGURE 12a. BURNING TIME FOR LUNAR DESCENT - $h_a$ = 50 km

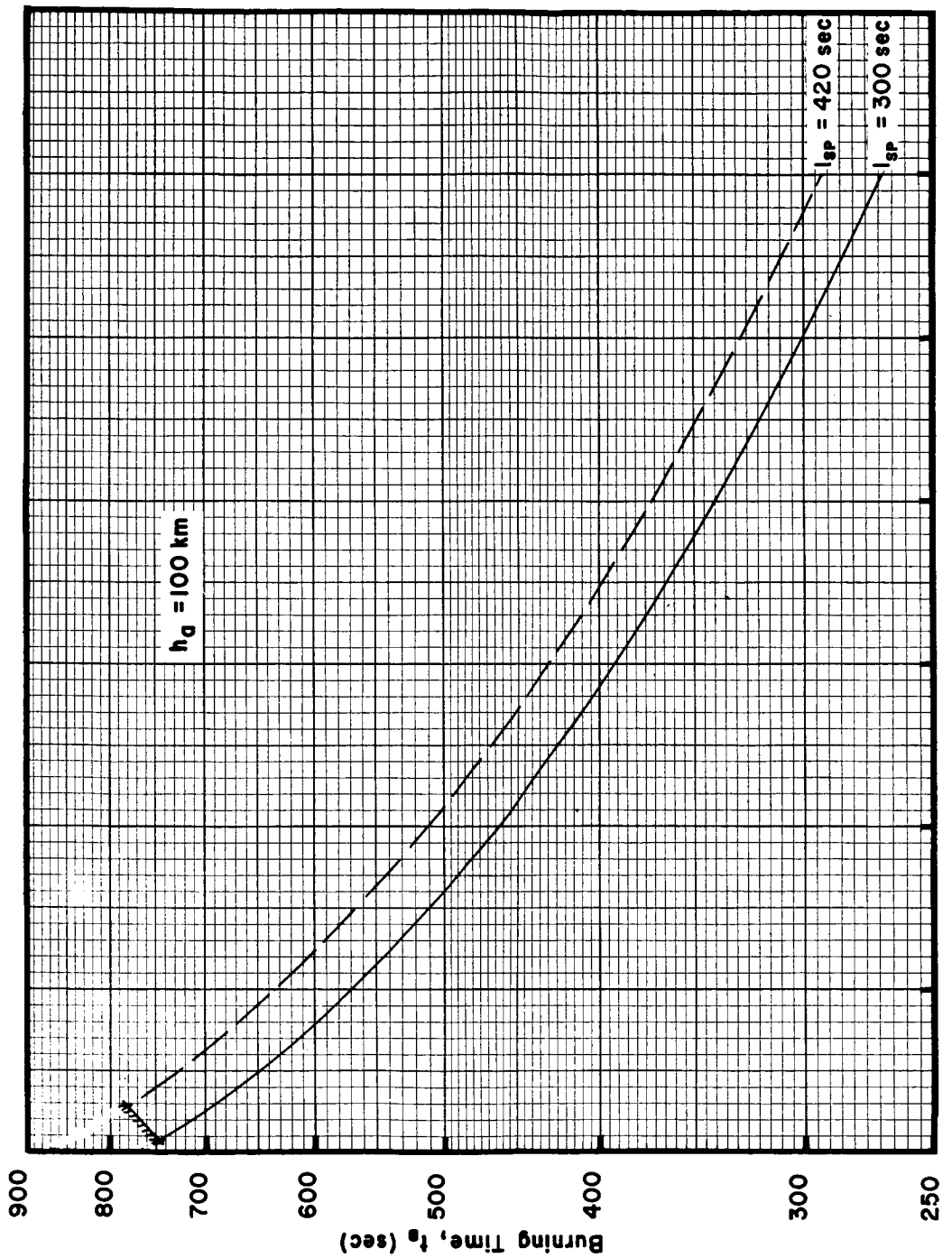



FIGURE 12b. BURNING TIME FOR LUNAR DESCENT - $h_a$ = 100 km

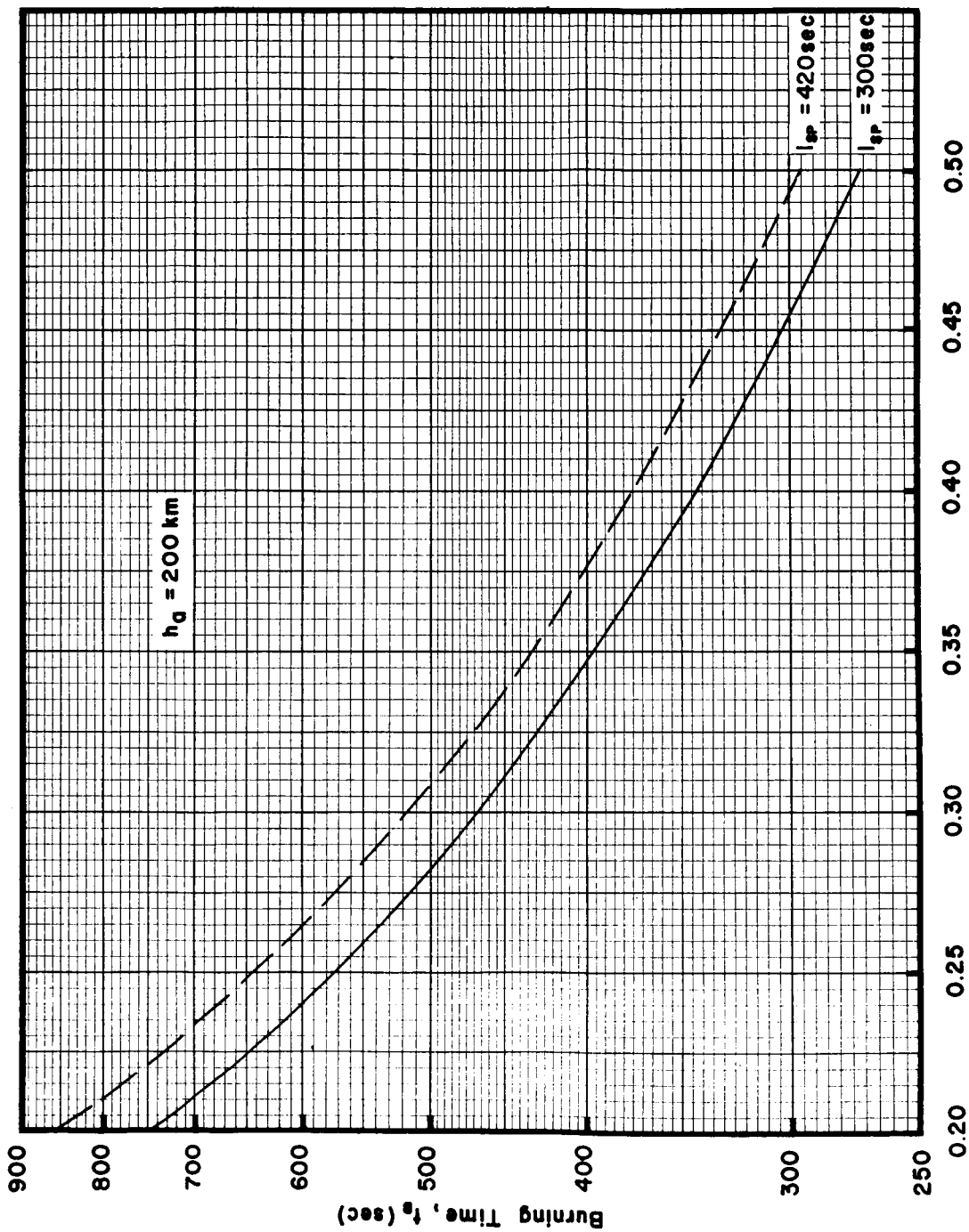



FIGURE 12c. BURNING TIME FOR LUNAR DESCENT - $h_a$ = 200 km

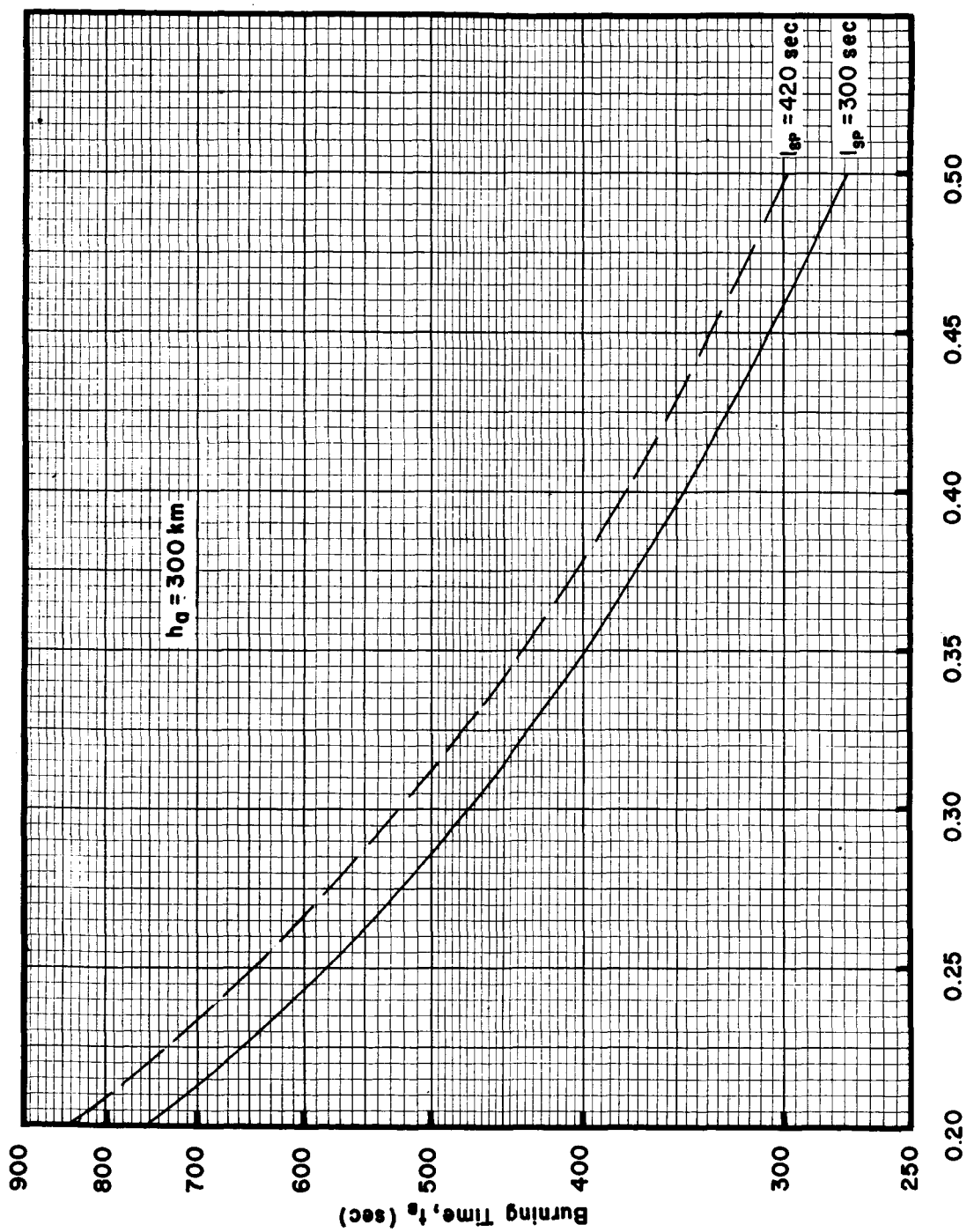



FIGURE 12d. BURNING TIME FOR LUNAR DESCENT - $h_a$ = 300 km

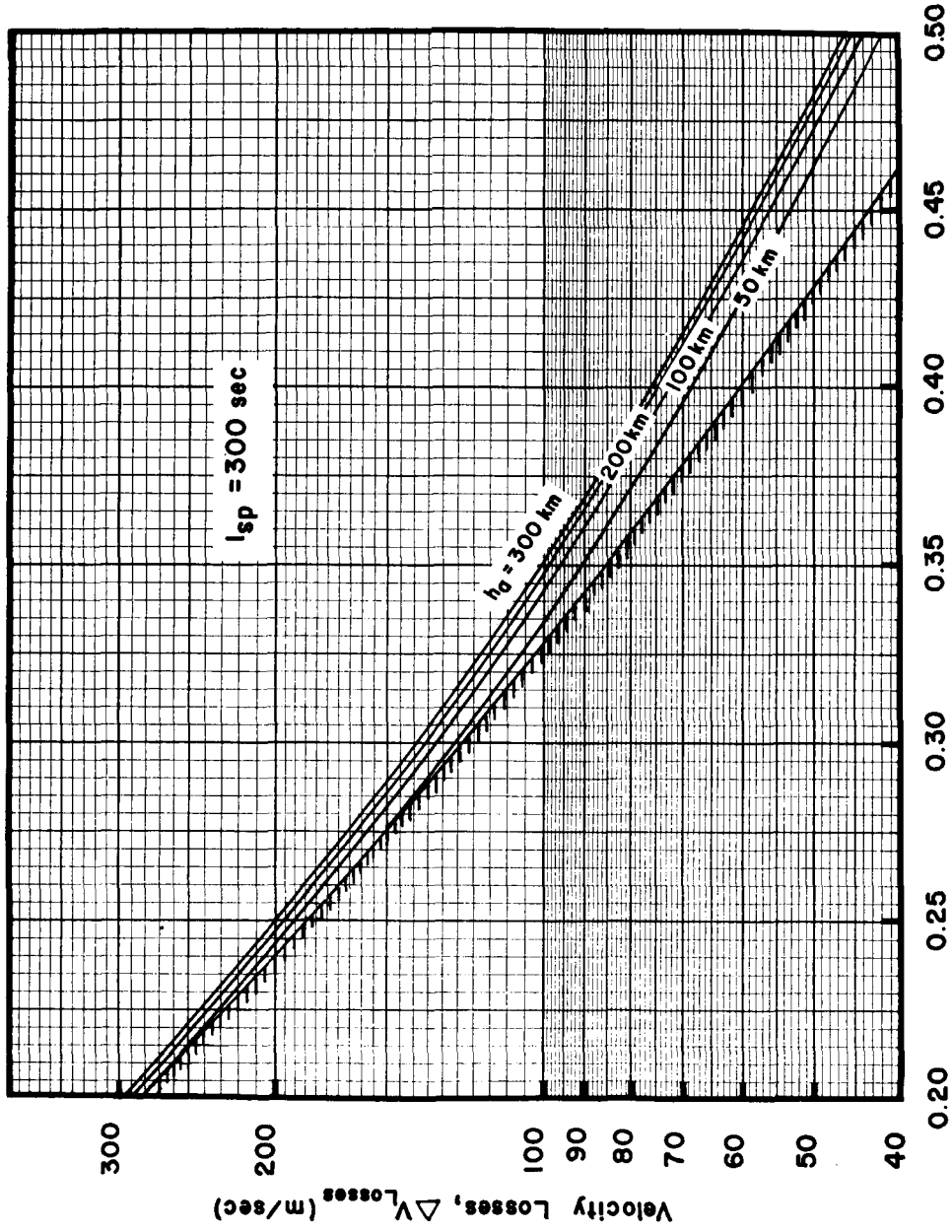



FIGURE 13a. VELOCITY LOSSES FOR LUNAR DESCENT - $I_{sp}$ = 300 sec.

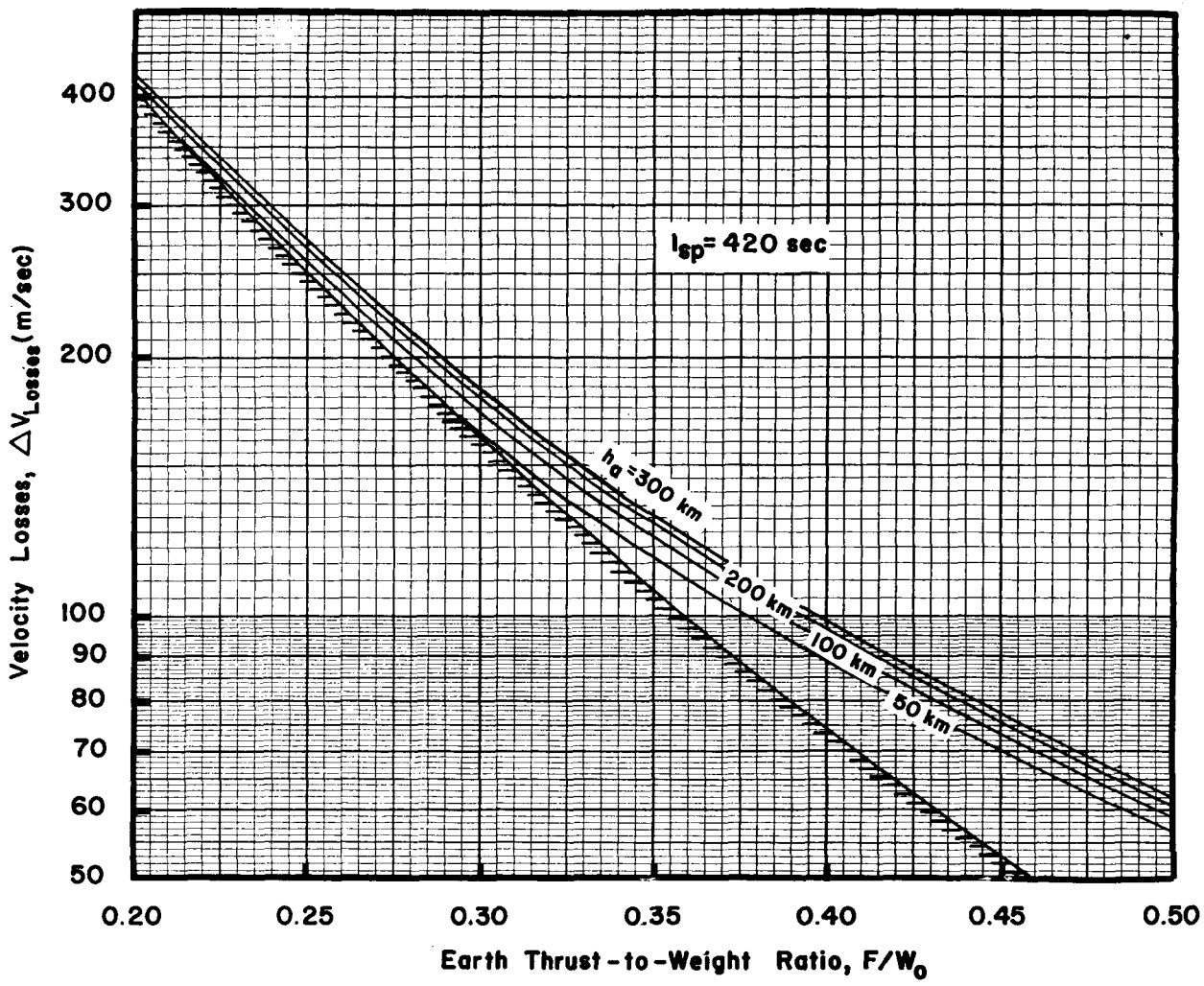



FIGURE 13b. VELOCITY LOSSES FOR LUNAR DESCENT - $I_{sp}$ = 420 sec.

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# PARAMETRIC PERFORMANCE ANALYSIS OF LUNAR MISSIONS

## PART II

## LUNAR DESCENT

By Charles M. Akridge and Sam H. Harlin

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